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ON CR-STRUCTURE AND F(2v+5,1) STRUCTURE SATISFYING $F^{2V+5}+F=0$

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Abstract-CR-submanifolds of a kahlerian manifold have been defined by A. Bejancu [1], and are now being studied by various authors, see [2] and [9]. The theory of f -structure was developed by Yano [10], Yano and Ishihara [11], Goldberg [6] and among others. The purpose of this paper is to show a relationship between CR- structures and $F(2\nu+5,1)$-structure satisfying

$$F^{2\nu+5}+F=0.$$

1. INTRODUCTION

Let F be a non-zero tensor field of the type (1, 1) and of class C^{∞} on an n-dimensional manifold M such that [7]

$$F^{2\nu+5}+F=0, \qquad 1.1$$

The rank of $(F)=r=$constant.
Let us define the operators on M as follows [7]

$$l=-F^{2\nu+4},$$
$$m=I+F^{2\nu+4}. \qquad 1.2$$

Where I denotes the identity operator. We will state the following two theorems[7]
Theorem 1.1. Let M be an $F(2\nu+5,1)$-structure manifold satisfying(1.1), then

$$l+m=I$$
$$l^2=l\,,m^2=m \qquad 1.3$$
$$\text{And } lm=ml=0$$

Thus for (1, 1) tensor field $F(\neq 0)$ satisfying (1.1), there exist complementary distributions D_l and D_m corresponding to the projection op- erators l and m respectively. Then, dim $D_l=r$ and dim $D_m=(n-r)$.

Theorem 1.2
We have,

a- $lF=Fl\,,mF=Fm=0$
b- $F^{2v+4}m=0$ 1.4
c- $F^{2v+4}l=-1$

Thus $F^{\nu+2}$ acts on D_l as an almost complex structure and on D_m as a null operator.

2. NIJENHUIS TENSOR

The Nijenhuis tensor $N(X,Y)$ of F satisfying (1.1) in M is expressed as follows for every vector field X, Y on M.

$$N(X,Y)=[FX,FY]-F[FX,Y]-F[X,FY]+F^2[X,Y] \qquad 2.1$$

Definition 2.1. If X, Y are two vector fields in M, then their lie bracket [X, Y] is defined by $[X,Y]=XY-YX$ (2.2)

3. CR-STRUCTURE

Let M be a differentiable manifold and T_cM be its complexified tangent bundle. A CR-structure on M is a complex subbundle H of T_cM such that $H_p\cap\bar{H}_p=0$ and H is involutive i.e. for complex vector fields X and Y in H, $[X,Y]$ is in H.

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3.1 CR-manifold

Let F-structure given by equation (1.1) be an integrable structure of rank $r = 2m$ on M. We define complex sub bundle H of T_cM by

$H_P = \{X - \sqrt{-1}FX, X \in \chi(D_l)\}$, where $\chi(D_l)$ is the $F(D_m)$ module of all differentiable sections of D_l

then $R_e(H) = D_l$ and $H_P \cap \bar{H}_p = 0$,

where $\bar{H}_p$ denotes the complex conjugate of H_P.

Theorem 3.1

If P and Q are two elements of H then the following relations holds

$$[P,Q] = [X,Y] - [FX,FY] - \sqrt{-1}([X,FY] + [FX,Y])$$

Proof. Let us define $P = X - \sqrt{-1}FX$ and $Q = Y - \sqrt{-1}FY$, then by direct calculation and on simplifying, we obtain

$$[P,Q] = \left[X - \sqrt{-1}FX, Y - \sqrt{-1}FY\right] = [X,Y] - [FX,FY] - \sqrt{-1}([X,FY] + [FX,Y]) \quad 3.1$$

Theorem 3.2

If $F(2\nu + 5, 1)$-structure satisfying equation (1.1) is integrable then we have

$$-F^{2\nu+3}([FX, FY] + F^2[X, Y]) = l([FX, Y] + [X, FY]). \quad 3.2$$

Proof. From equation (2.1), we have

$$N(X, Y) = [FX, FY] - F[FX, Y] - F[X, FY] + F^2[X, Y].$$

Since $N(X, Y) = 0$, we obtain

$$[FX, FY] + F^2[X, Y] = F([FX, Y] + [X, FY]. \quad 3.3$$

Operating (3.3) by $(-F^{2\nu+3})$, we get

$$(-F^{2\nu+3})([FX, FY] + F^2[X, Y]) = (-F^{2\nu+4})([FX, Y] + [X, FY])$$

In view of equation (1.2) in the above equation, we obtain (3.2), which proves the theorem.

Theorem 3.3

The following identities hold

$$mN(X, Y) = m[FX, FY]. \quad 3.4$$

$$mN(F^{2\nu+3}X, Y)$$
$$= m[F^{2\nu+4}X, FY]. \quad 3.5$$

Proof. The proof of equations (3.4) and (3.5) follows easily by virtue of theorems 1.1, 1.2 and equation (2.1).

Theorem3.4

For any two vector fields X and Y, the following con- ditions are equivalent

a. $mN(X, Y) = 0$,
b. $m[FX, FY] = 0$,
c. $mN(F^{2\nu+3}X, Y) = 0$,
d. $m[F^{2\nu+4}X, FY] = 0$,
e. $m[F^{2\nu+4}lX, FY] = 0$.

Proof. In consequence of equations (1.1), (1.2), (2.1) and theorems 1.2, 3.3, the above identities can be proved to be equivalent.

Theorem3.5

If $F^{\nu+2}$ acts on D_l as an almost complex structure, then

$$m[F^{\nu+2}lX, FY] = m[\sqrt{-1}X, FY] = 0. \quad 3.6$$

Proof. In view of equations (1.4a), (1.4b), we see that $F^{\nu+2}$ acts on D_l as an almost complex structure then equation (3.6) follows in an obvious manner. To show that $m[F^{\nu+2} lX, FY] = 0$, we use the definition 2.1 and in view of equation (1.4 a), the result follows directly.'

Theorem3.6

For $X, Y \in \chi(D_l)$, we have

$$l([X, FY] + [FX, Y]) = [X, FY] + [FX, Y]$$

Proof. Since $[X, FY]$ and $[FX, Y] \in \chi(D_l)$, on making use of (1.4a) and definition 2.1 we obtain the result.

Theorem3.7

The integrable $F(2\nu + 5, 1)$-structure satisfying (1.1) on M defines a CR-structure H on it such that $R_e H = D_l$.

Proof. In view of the fact that $[X, FY]$ and $[FX, Y] \in \chi(D_l)$ and on using equations (3.1), (3.2) and theorem 3.6, we have $[P, Q] \in \chi(D_l)$. Then $F(2\nu + 5, 1)$-structure satisfying (1.1) on M defines a CR-structure.

Definition 3.8

Let $\widetilde{K}$ be the complementary distribution of $R_e(H)$ to TM. We define a morphism of vector bundles $F: TM \to TM$ given by $F(X) = 0\ \forall\ X \in \chi F(\widetilde{K})$ such that

$F(X) = \frac{1}{2}\sqrt{-1}(P - \overline{P})$ where $P = X + \sqrt{-1}Y, Y \in X(H_P)$ and $\bar{P}$ is complex conjugate of P.

Corollary3.9.[3] If $P = X + \sqrt{-1}Y$ and $\bar{P} = X - \sqrt{-1}Y$ belong to H_P and

$$F(X) = \frac{1}{2}\sqrt{-1}(P - \overline{P}), F(Y) = \frac{1}{2}\sqrt{-1}(P + \overline{P})$$

And $F(-Y) = -\frac{1}{2}(P + \overline{P})$ then $F(X) = -Y, F^2(X) = -X$ and $F(-Y) = -X$

Theorem 3.10

If M has a CR-structure H, then we have $F^{2\nu+5} + F = 0$ and consequently $F(2\nu + 5, 1)$-structure satisfying (1.1) is defined on M such that the distributions D_l and D_m coincide with Re(H) and $\widetilde{K}$ respectively.

Proof .Suppose M has a CR-structure. Then in view of definition 3.8 and corollary 3.9, we have

$$F(X) = -Y. \qquad 3.8$$

Operating (3.8) by F^{2K} we get

$$F^{2\nu}(F(X) = F^{2\nu}(-Y) \qquad 3.9$$

We can write the right hand side of (3.9) as follows

$$F^{2\nu+1}(X) = F^{2\nu-1}(F(-Y) \qquad 3.10$$

On making use of corollary 3.9, the above equation becomes

$$F^{2\nu+1}(X) = F^{2\nu-1}(-X)$$
$$= -F^{2\nu-1}(X), \qquad 3.11$$

which can be written as

$$F^{2\nu+1}(X) = -F^{2\nu-2}(F(X))$$
$$= -F^{2\nu-2}(-Y)$$
$$= F^{2\nu-2}(Y) \qquad 3.12$$

We continue simplifying in this manner and obtain

$$F^{2\nu+1}(X) = -F(X) \qquad 3.13$$

i.e

$$F^{2\nu+1}(X) + F(X) = 0. \qquad 3.14$$

Similarly we have $F^{2\nu+3}(X) = F^{2\nu+1}(-X)$

$$= -F^{2\nu+1}(X) \quad 3.15$$

i.e $$F^{2\nu+3}(X) = -F^{2\nu}(F(X))$$

$$= -F^{2\nu}(-Y) \quad 3.16$$

$$= F^{2\nu}(Y)$$

We continue simplifying in this manner and obtain

$$F^{2\nu+3}(X) = -F(X) \quad 3.17$$

$$F^{2\nu+3}(X) + F(X) = 0. \quad 3.18$$

Again, we continue simplifying in this manner and obtain,

$$F^{2\nu+5}(X) + F(X) = 0. \quad 3.19$$

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